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## Heavy Ion $e^+e^-$ Pairs to All Orders in $Z\alpha$

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**Abstract.** The heavy ion cross section for continuum  $e^+e^-$  pair production has been calculated to all orders in  $Z\alpha$ . Comparison is made with available CERN SPS and RHIC STAR data. Computed cross sections are found to be reduced from perturbation theory with increasing charge of the colliding heavy ions and for all energy and momentum regions investigated. Au or Pb total cross sections are reduced by 28% (SPS), 17% (RHIC), and 11% (LHC). For very high energy ( $E_{e^+}, E_{e^-} > 3$  GeV) forward pairs at LHC the reduction from perturbation theory is a bit larger (17%). Use of zero degree calorimeter triggering (and thus small impact parameter weighting) makes impact parameter representation of exact pair production useful. Preliminary exact calculations in the zero impact parameter limit show a much larger reduction from perturbation theory (about 40%) at both RHIC and LHC.

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### 1. Introduction

A two center light cone calculation of continuum pairs can be carried out by exactly solving the semi-classical Dirac equation[ 1, 2, 3, 4] for colliding  $\delta$  function potentials

$$V(\boldsymbol{\rho}, z, t) = \delta(z - t)(1 - \alpha_z)\Lambda^-(\boldsymbol{\rho}) + \delta(z + t)(1 + \alpha_z)\Lambda^+(\boldsymbol{\rho})$$

in the collider center of mass (lab) frame, where

$$\Lambda^\pm(\boldsymbol{\rho}) = -Z\alpha \ln \frac{(\boldsymbol{\rho} \pm \mathbf{b}/2)^2}{(b/2)^2}.$$

Baltz and McLerran[ 2] originally noted an agreement with perturbation theory in the exact result. Segev and Wells[ 3] noted this agreement and also noted the

scaling with  $Z_1^2 Z_2^2$  seen in SPS data for 160 GeV/c Pb ions on C, Al, Pa, Au and for 200 GeV/c S ions on C, Al, Pa, Au. The experimental group, Vane et al., summarized their data: “Cross sections scale as the product of the squares of the projectile and target nuclear charges[ 5].”

On the other hand, photoproduction on a heavy target shows a negative correction proportional to  $Z^2$ [ 6]. Several authors have argued that a correct regularization of the exact Dirac equation amplitude should lead to Coulomb corrections[ 7, 8].

## 2. Cross Section with Higher Order Coulomb Corrections

I have previously showed how a physical cutoff of the transverse potential  $\Lambda^\pm(\rho)$  leads to Coulomb corrections[ 9] consistent with the Lee and Milstein approximate analytical result[ 8]. In this section I present recent “exact” Dirac equation calculations of total cross sections that make use of this physical cutoff[ 10].

Table 1 shows the results of exact numerical calculations indicating reductions from perturbation theory. In addition, calculations of positron transverse and longitudinal spectra show that the reductions persist to the highest and lowest momentum values.

Note that the total cross section at CERN SPS energy is reduced from perturbation theory by 28%, a disagreement with the experimentally presented perturbative scaling. Nevertheless, given the difficulty of the SPS experiment as described by the authors, I would argue that the apparent lack of Coulomb corrections in the data needs to be verified in other ultrarelativistic heavy ion experiments.

**Table 1.** Computer calculations compared with analytical formula results.  $\gamma$  is defined for one of the ions in the frame of equal magnitude and opposite direction velocities. Total cross sections are expressed in barns.

		Exact	Perturb.	Difference
Pb + Au $\gamma = 9.2$ SPS	Computer Evaluation[ 10]	2670	3720	-1050
	Racah Formula[ 11]		3480	
	Lee-Milstein[ 8]	3050	5120	-2070
Au + Au $\gamma = 100$ RHIC	Computer Evaluation	28,600	34,600	-6,000
	Racah Formula		34,200	
	Lee-Milstein	34,100	42,500	-8,400
	Hencken, Trautman, Baur[ 12]		34,000	
Pb + Pb $\gamma = 2960$ LHC	Computer Evaluation	199,000	224,000	-25,000
	Racah Formula		226,000	
	Lee-Milstein	226,000	258,000	-32,000

### 3. RHIC STAR Data

$e^+e^-$  pair production accompanied by nuclear dissociation has been measured by STAR. Comparison with perturbative QED calculations allowed a limit to be set “on higher-order corrections to the cross section,  $-0.5\sigma_{QED} < \Delta\sigma < 0.2\sigma_{QED}$ , at a 90% confidence level[ 13].”

Calculations in the STAR acceptance without dissociation provide an indication of the relative difference between perturbation theory and the exact result. In the STAR acceptance the exact result is calculated to be 17% lower than perturbation theory. This rough estimate,  $\Delta\sigma = -0.17\sigma_{QED}$ , is not excluded by STAR.

### 4. Forward Pairs at LHC

A sample numerical calculation has been performed using the same method for  $e^+e^-$  production by Pb + Pb ions with cuts from a possible detector setup at LHC. With electron and positron energy  $E$  and angle  $\theta$  in the range,  $3 \text{ GeV} < E < 20 \text{ GeV}$  and  $.00223 \text{ radians} < \theta < .00817 \text{ radians}$ , the no form factor perturbation theory cross section of 2.88 b is reduced by 18% to 2.36 b in an exact numerical calculation.

If forward  $e^+e^-$  pairs are to be employed for luminosity measurements at LHC, then it seems necessary to consider the Coulomb corrections to the predicted cross sections.

### 5. Probabilities at Small Impact Parameter: the Zero Impact Parameter Limit

Zero degree calorimeter (ZDC) triggering (e.g. for the STAR pair production) weights smaller parameter contributions: the probability at each impact parameter goes as the product of the dissociation probability (ZDC) and the pair production probability. Pair production probability as a function of impact parameter is needed to describe ZDC triggered events as was done for  $\rho$  production at STAR[ 14]. I calculate the number weighted probability  $P_T$  (or number operator),  $P_T = \sum_{n=1}^{\infty} nP_n(b)$ , for producing  $e^+e^-$  pairs at some impact parameter  $b$ . As a first step I compute the  $\mathbf{b} = 0$  limit.

For Au + Au at RHIC the perturbation theory result is  $P^0(0) = 1.64$ , the exact result  $P(0) = .94$ , or  $P(0) = .57P^0(0)$ . This limit may have some relevance to the very high energy pairs measured by STAR which necessarily come from relatively low impact parameters. Again  $-0.5\sigma_{QED} < \Delta\sigma < 0.2\sigma_{QED}$  from STAR is not contrary the indications from the exact result in this limit.

For Pb + Pb at LHC the perturbation theory result is  $P^0(0) = 4.07$ , and the exact result is  $P(0) = 2.39 = .59P^0(0)$ .

## 6. Conclusions

A full numerical evaluation of the “exact” total cross section for  $e^+e^-$  production with gold or lead ions shows reductions from perturbation theory of 28% (SPS), 17% (RHIC), and 11%(LHC). Reductions are 43% (RHIC), and 41%(LHC) for  $b = 0$ . For large  $Z$  no final momentum region was found in which there was no reduction or an insignificant reduction of the exact cross section.

## 7. For the Future

The predicted reduction of continuum  $e^+e^-$  pair production from  $Z_A^2 Z_B^2$  scaling at higher  $Z$  has never been observed experimentally. An obvious suggestion for experiment is to compare electromagnetic  $e^+e^-$  pair production in Au + Au with e.g. Ca + Ca at RHIC or LHC.

There is also a clear need to construct a computer program to calculate  $P(b)$  exactly for  $e^+e^-$  to all order in  $Z\alpha$ .

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